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# Top quark production in $e^+e^-$ annihilation*

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**Abstract.** We analyze the four-fermion reactions $e^+e^- \to 4f$ containing a single top quark and three other fermions, a possible decay product of the resonant anti-top quark, in the final state. This allows us to estimate the contribution of the nonresonant Feynman graphs and effects related to the off mass shell production and decay of the top quark. We test the sensitivity of the total cross section at center of mass energies in the $t\bar{t}$ threshold region and far above it to the variation of the top quark width. We perform the calculation in an arbitrary linear gauge in the framework of the standard model and discuss the important issue of gauge symmetry violation by a constant top quark width.

## 1 Introduction

The production of the top quark in $e^+e^-$ annihilation is an issue which has attracted a lot of interest of both experimenters and theorists over the past decade. In the present decade the interest will certainly be growing in the prospect of new high energy $e^+e^-$ colliders, gradually moving from the stage of general discussion to detailed planning, and hopefully to the stage of construction and successful operation [1]. As the $e^+e^-$ machines operate in a very clean experimental environment, they provide a unique possibility of precise measurement of the top quark's physical properties which, as many expect, may go beyond the standard model (SM) and give decisive hints for the development of new physical ideas. In order to disentangle the possible effects of new physics from the standard physics, it is crucial to know the SM predictions as precisely as possible.

It has taken a lot of efforts to obtain precise SM predictions for the top pair production in the threshold region. A substantial improvement of the convergence of the perturbation series has been achieved by computing the next-to-next-to-leading order corrections to the top quark pair production cross section near threshold [2] and understanding the renormalon cancellation mechanism [3]. The $\mathcal{O}(\alpha\alpha_s)$ corrections to the top decay into a $W$ boson and a $b$ quark are also known [4].

In the present note, we concentrate on the effects related to the fact that one of the quarks in the $t\bar{t}$ pair may be produced off mass shell. In particular, we let the anti-top quark $\bar{t}$ decay into a final state possible in the framework of the SM, i.e., we consider reactions

$$e^+e^- \to t\bar{b}f\bar{f}', \tag{1}$$

where $f = e^-, \mu^-, \tau^-, d, s$ and $f' = \nu_e, \nu_\mu, \nu_\tau, u, c$, respectively, taking into account the complete set of Feynman graphs which contribute to the specific final state at the tree level. We pay attention to the very important issue of gauge symmetry breaking caused by the nonzero constant widths of unstable particles, in particular that of the top quark, which are kept as free parameters. We also test the sensitivity of the total cross sections of (1) to the variation of the top width. The deviation of the top width from its SM value may indicate new physics. In order to test the reliability of our results, we perform the calculation in arbitrary linear $R_\xi$ gauge. We neglect radiative corrections, the correct treatment of which demands an extra effort and is beyond the scope of the present work.

We describe the basics of the calculation in the next section. Our results are presented and discussed in Sect. 3 and, finally, in Sect. 4, we give our concluding remarks.

## 2 Calculation in arbitrary linear gauge

The calculation of the necessary matrix elements relies on the method proposed in [5] and further developed in [6]. As in [6], fermion masses are kept nonzero both in the matrix elements and in the kinematics. Among others, this has the advantage that the Higgs boson effects can be incorporated consistently and the pole related to the photon exchange in the $t$-channel can be handled better than in the massless fermion case.

In order to estimate the gauge symmetry violation effects related to the nonzero widths of unstable particles, we perform the calculation in two different schemes: the "fixed widths scheme" (FWS) and in the so called "complex mass scheme" (CMS) of [7]. Both schemes introduce

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the constant particle widths through the complex mass parameters:

$$M_V^2 = m_V^2 - \mathrm{i} m_V \Gamma_V,\ V = W, Z, H,\ M_t = m_t - \mathrm{i}\Gamma_t/2, \quad (2)$$

which replace the masses in the corresponding propagators. The coupling constants are given in terms of the electric charge and the electroweak mixing parameter $\sin^2\theta_{\mathrm{W}}$. In FWS, the electroweak mixing parameter is kept real, i.e., it is given by

$$\sin^2\theta_{\mathrm{W}} = 1 - m_W^2/m_Z^2, \quad (3)$$

where $m_W$ and $m_Z$ are the physical masses of the $W^\pm$ and $Z^0$ boson. In CMS, $\sin^2\theta_{\mathrm{W}}$ is given in terms of the complex masses $M_W$ and $M_Z$ of (2) by

$$\sin^2\theta_{\mathrm{W}} = 1 - M_W^2/M_Z^2; \quad (4)$$

thus it is a complex number. The CMS has the advantage that it preserves the Ward identities [7], provided all the fermion widths are zero. We have tested the gauge invariance numerically for all the reactions (1) under the assumption of the zero top quark widths. With the nonzero top width introduced in the Feynman propagator of the top quark,

$$\mathrm{i}S_t^F(p) = \mathrm{i}\frac{\not p + M_t}{p^2 - M_t^2}, \quad (5)$$

the Ward identities are not satisfied anymore and hence the gauge symmetry is violated.

There has been a lot of discussion of the gauge invariance issue in the literature of the past few years [8]. The best way to solve the problem, however involved and complicated it might be, is to work in the "fermion-loop scheme" (FLS), which resumes higher order effects coming from one-loop fermion contributions to bosonic propagators together with parts of the vertices necessary for keeping the corrections gauge invariant. Unfortunately, a similar Dyson resummation has not been worked out in a detailed way for the propagator of an unstable fermion up to now. Therefore, the substitution of (2) has no full theoretical justification in the framework of quantum field theory. However, a comparison of numerical results obtained in the schemes using fixed widths in both the $s$- and $t$-channel gauge boson propagators with those derived within FLS, which show no numerically relevant discrepancies for several four-fermion reactions and the corresponding bremsstrahlung processes, speaks in favor of the simplified approach based on substitution of (2) in each propagator of an unstable particle. Therefore, in the following we will restrict ourselves to this simplified approach.

In order to quantitatively estimate gauge symmetry violation effects induced by the substitution of (2), we perform the calculation of the matrix elements in the linear gauge with arbitrary real gauge parameters $\xi_V, V = \gamma, W, Z$. We then vary the parameters in a very wide range from $\xi_V = 1$ corresponding to the 't Hooft–Feynman gauge (FG) to $\xi_V = 10^{16}$ which, in the double precision of the Fortran programming language, corresponds to the unitary gauge (UG). We would like to stress at this point that we take into account contributions from the exchange of the would-be Goldstone bosons in the $R_\xi$ gauge, which are absent in the unitary gauge. If the change in the cross section induced by the change in gauge parameters is smaller than the accuracy of the Monte Carlo integration, we assume that the gauge violation effects induced by the nonzero top width are numerically irrelevant and that we may consider the corresponding results trustworthy. On the other hand, if the results depend on the choice of the gauge parameters they are useless, but we may try to reduce the dependence by imposing cuts on the phase space integration. This simple prescription, however doubtful from the purely theoretical point of view it might be, allows one to treat the particle widths as independent parameters and test the reliability of the numerical results in an efficient way.

The phase space integration is performed numerically using a multichannel Monte Carlo (MC) approach and the integration routine VEGAS [10]. The 7 dimensional phase space element of reaction (1) is parameterized in a few different ways in order to account for the most relevant peaks of the matrix elements: the $\sim 1/t$ pole caused by the $t$-channel photon exchange, the Breit–Wigner shape of the $W^\pm$ and $Z^0$ resonances, the $\sim 1/s$ behavior of light fermion pair production, and the $\sim 1/t$ pole due to the neutrino exchange at the same time.

## 3 Numerical results

In this section, we will present numerical results for all the four-fermion channels of the reaction (1) possible in the SM.

We define the SM physical parameters in terms of the gauge boson masses and widths, the top mass and the Fermi coupling constant. We take the actual values of the parameters from [9]:

$$\begin{aligned} m_W &= 80.419\,\mathrm{GeV}, \\ \Gamma_W &= 2.12\,\mathrm{GeV}, \\ m_Z &= 91.1882\,\mathrm{GeV}, \\ \Gamma_Z &= 2.4952\,\mathrm{GeV}, \\ m_t &= 174.3\,\mathrm{GeV}, \\ G_\mu &= 1.16639\times 10^{-5}\,\mathrm{GeV}^{-2}. \end{aligned} \quad (6)$$

We assume the Higgs boson mass to be $m_H = 115\,\mathrm{GeV}$ and, if not stated otherwise, the top quark width is taken to be $\Gamma_t = 1.5\,\mathrm{GeV}$.

For the sake of definiteness we also list the other fermion masses we use in the calculation [9]:

$$\begin{aligned} &m_e = 0.510998902\,\mathrm{MeV}, \quad m_\mu = 105.658357\,\mathrm{MeV}, \\ &m_\tau = 1777.03\,\mathrm{MeV}, \\ &m_u = 5\,\mathrm{MeV}, \quad m_d = 9\,\mathrm{MeV}, \quad m_s = 150\,\mathrm{MeV}, \\ &m_c = 1.3\,\mathrm{GeV}, \quad m_b = 4.4\,\mathrm{GeV}. \end{aligned} \quad (7)$$

We neglect Cabibo–Kobayashi–Maskawa mixing, i.e., we assume the CKM matrix to be a unit matrix.

**Table 1.** Cross sections in fb of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ at different center of mass energies in different schemes, CMS and FWS, and gauges, UG and FG. The numbers in parentheses show the uncertainty of the last decimals

| $\sqrt{s}$ (GeV) | $\sigma_{\rm CMS}^{\rm UG}$ | $\sigma_{\rm CMS}^{\rm FG}$ | $\sigma_{\rm FWS}^{\rm UG}$ | $\sigma_{\rm FWS}^{\rm FG}$ |
|---|---|---|---|---|
| 190 | $2.6174(7) \times 10^{-8}$ | $2.6174(7) \times 10^{-8}$ | $2.6185(7) \times 10^{-8}$ | $2.6185(7) \times 10^{-8}$ |
| 340 | 0.7837(4) | 0.7837(4) | 0.7839(4) | 0.7840(4) |
| 360 | 41.27(10) | 41.27(10) | 41.28(10) | 41.29(10) |
| 500 | 60.06(13) | 60.04(13) | 59.75(30) | 59.90(29) |
| 2000 | 5.59(3) | 5.56(3) | 5.51(7) | 5.51(8) |

**Table 2.** Cross sections in fb of $e^+e^- \to t\bar{b}e^-\bar{\nu}_e$ in the CMS in two different gauges, UG and FG, and for two different cuts on the electron angle with respect to the beam

| $\sqrt{s}$ | $5^0 < \theta(e^-, \text{beam}) < 175^0$ | | $1^0 < \theta(e^-, \text{beam}) < 179^0$ | |
|---|---|---|---|---|
| (GeV) | $\sigma_{\rm CMS}^{\rm UG}$ | $\sigma_{\rm CMS}^{\rm FG}$ | $\sigma_{\rm CMS}^{\rm UG}$ | $\sigma_{\rm CMS}^{\rm FG}$ |
| 190 | $0.6607(4) \times 10^{-5}$ | $0.6607(4) \times 10^{-5}$ | $0.10520(4) \times 10^{-4}$ | $0.10531(4) \times 10^{-4}$ |
| 340 | 0.7993(4) | 0.7993(4) | 0.8251(4) | 0.8253(4) |
| 360 | 41.21(11) | 41.20(11) | 41.32(8) | 41.32(8) |
| 500 | 59.78(15) | 59.75(15) | 60.16(15) | 60.19(15) |
| 2000 | 6.81(3) | 6.82(3) | 7.97(3) | 8.00(3) |

The fine structure constant is calculated from

$$\alpha_W = \sqrt{2} G_\mu m_W^2 \sin^2\theta_{\rm W}/\pi, \tag{8}$$

with the real electroweak mixing parameters of (3) in both schemes FWS and CMS.

Except for the check of gauge invariance discussed in the previous section, we perform a few other checks. Our results reproduce those of [6] for a top mass smaller than $m_W$ and the zero top width. The corresponding matrix elements in the absence of the Higgs boson exchange has been checked against MADGRAPH [11]. The phase space generation routine for particles of large masses has been written in two independent ways.

In Table 1, we show the results for the cross sections of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ at different center of mass energies obtained in different schemes and gauges: the complex mass scheme (CMS), the fixed width scheme (FWS), the unitary gauge (UG) and the Feynman gauge (FG). We have integrated over the full four particle phase space without any cuts. We can see that the results hardly depend on the gauge choice both in the CMS and FWS. Actually, they nicely agree with each other within one standard deviation of the MC integration.

In Table 2, we present the results for $e^+e^- \to t\bar{b}e^-\bar{\nu}_e$ obtained in the CMS and in two different gauges, UG and FG. In order to reduce the dependence on the gauge choice induced by the nonzero top width, we have imposed a cut on the electron angle with respect to the beam $\theta(e^-, \text{beam})$. Again there is a rather small dependence on the cut for the energies presented in Table 2. From a comparison with the corresponding numbers of Table 1, we can infer that the $t$-channel Feynman graphs of reaction $e^+e^- \to t\bar{b}e^-\bar{\nu}_e$ do not contribute much to the total cross section in the presence of the cut on the final electron angle. When we reduce the cut further so that the denominator of the $t$-channel photon propagator becomes of the order of the electron mass squared, the dependence on the gauge becomes substantial and the results are meaningless.

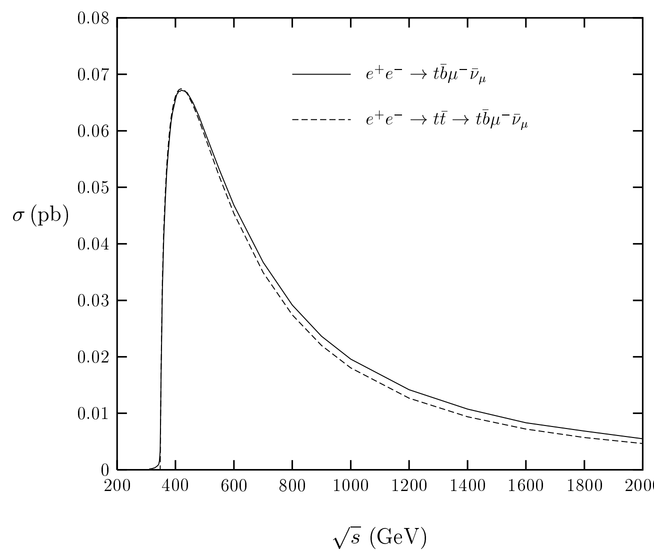


**Fig. 1.** The energy dependence of the total cross sections of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ and $e^+e^- \to t\bar{t} \to t\bar{b}\mu^-\bar{\nu}_\mu$

The results for the channels of reaction (1) which do not contain an electron in the final state are shown in Table 3. They were obtained in the CMS and unitary gauge.

In Fig. 1, we show the energy dependence of the total cross section of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ calculated with the complete set of Feynman graphs and the approximate cross section $e^+e^- \to t\bar{t} \to t\bar{b}\mu^-\bar{\nu}_\mu$. The latter has been ob-

**Table 3.** Cross sections in the CMS in fb of different channels of reaction (1) not containing a final state electron

| Channel of reaction (1) | $\sqrt{s}$ (GeV) 190 | 340 | 360 | 500 | 2000 |
|---|---|---|---|---|---|
| $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ | $2.6174(7) \times 10^{-8}$ | 0.7837(4) | 41.3(1) | 59.8(3) | 5.42(7) |
| $e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau$ | $1.9331(4) \times 10^{-8}$ | 0.7831(4) | 41.2(1) | 59.6(3) | 5.47(7) |
| $e^+e^- \to t\bar{b}d\bar{u}$ | $7.880(2) \times 10^{-8}$ | 2.351(1) | 123.8(3) | 179.9(9) | 16.3(2) |
| $e^+e^- \to t\bar{b}s\bar{c}$ | $6.616(2) \times 10^{-8}$ | 2.350(1) | 123.8(3) | 178.9(9) | 16.7(2) |

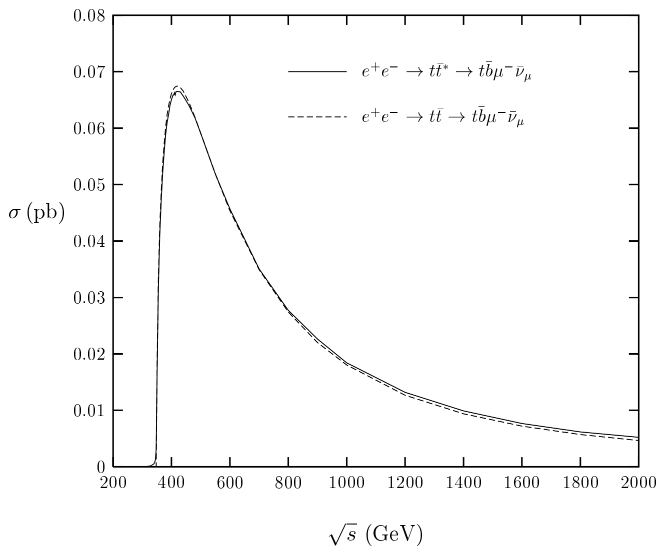


**Fig. 2.** The energy dependence of the total cross sections of $e^+e^- \to t\bar{t}^* \to t\bar{b}\mu^-\bar{\nu}_\mu$ and $e^+e^- \to t\bar{t} \to t\bar{b}\mu^-\bar{\nu}_\mu$

tained by multiplying the on shell top pair production cross section by the corresponding three body top decay width

$$\sigma(e^+e^- \to t\bar{t} \to t\bar{b}\mu^-\bar{\nu}_\mu) = \sigma(e^+e^- \to t\bar{t})\Gamma(\bar{t} \to \bar{b}\mu^-\bar{\nu}_\mu). \quad (9)$$

We have taken over the SM part of the analytic formula for the width $\Gamma(\bar{t} \to \bar{b}\mu^-\bar{\nu}_\mu)$ with massless final state fermions from [12]. In the calculation of $\sigma(e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu)$ we have used the FWS scheme and neglected the Higgs boson contribution. We see that (9) approximates the complete tree level calculation well, not only just above the threshold but also for higher center of mass energies. The relative difference between the both results is 3.5% at 360 GeV, 1.3% at 500 GeV and −5.0% at 800 GeV. This nice agreement is somewhat amazing as, except for one Feynman graph which contains a resonant top propagator, there are nine nonresonant graphs which contribute to $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ in the unitary gauge.

The explanation of this fact can easily be found if one looks at Fig. 2, where we have plotted, against the center of mass energy, the cross section of (9) and another approximated cross section obtained by integrating over the full four particle phase space the squared matrix element containing only the top resonant Feynman graph.

**Table 4.** Cross sections in fb of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ for different values of the top quark width. The calculation has been performed in the CMS and UG

| $\sqrt{s}$ (GeV) | $\Gamma_t$ (GeV) 1.5 | 1.6 | 1.7 |
|---|---|---|---|
| 190 | $2.6174(7) \times 10^{-8}$ | $2.6186(4) \times 10^{-8}$ | $2.6186(4) \times 10^{-8}$ |
| 340 | 0.7837(4) | 0.7832(3) | 0.7830(3) |
| 360 | 41.27(10) | 38.65(7) | 36.31(6) |
| 500 | 60.06(13) | 56.37(13) | 53.13(12) |
| 2000 | 5.59(3) | 5.30(2) | 5.09(2) |

The small discrepancy between the two curves in Fig. 2 is a measure of spin correlations and off-shellness of the $\bar{t}$ quark.

We illustrate the dependence of the total cross section of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ on the top quark width $\Gamma_t$ in Table 4. We see that the cross section at $s^{1/2} = 360$ GeV, i.e. just above the threshold, is almost exactly proportional to $1/\Gamma_t$. This kind of dependence holds also at $s^{1/2} = 500$ GeV, which is already much above the threshold and it survives almost unaltered at $s^{1/2} = 2$ TeV. This means that the cross section of $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$ is well approximated by the resonant $\bar{t}$ production and its subsequent decay. This kind of dependence offers a new way of measurement of the top quark width, alternative to the measurement based on the shape of the $t\bar{t}$ threshold [13].

## 4 Summary and outlook

We have analyzed the top quark production in $e^+e^-$ annihilation at a new high luminosity linear collider like TESLA. We have estimated the contribution of the nonresonant Feynman graphs and effects related to the off mass shell production and decay of one of the top quarks. Those effects are typically of the order of a few percent. Therefore, one should take them into account in the analysis of the future data. We have shown that the cross section of reaction (1) is dominated by the resonant $\bar{t}$ production and its subsequent decay not only at center of mass energies in the $t\bar{t}$ threshold region but also far above it. We have tested the sensitivity of the total cross sections to the variation of the top quark width and confirmed expected proportionality to $1/\Gamma_t$ over a very wide energy range beginning from the threshold. This kind of dependence offers an alternative way of measurement of the top quark width. By performing the calculation in an arbitrary linear gauge in the framework of the standard model we have been able to address the important issue of gauge symmetry violation by the constant top quark width.

It would be desirable to consider the effects related to the off shell production of the second quark of the $t\bar{t}$ pair and to include leading radiative corrections in the analysis of the future data.

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